BAUMOL'S MODEL FOR MANAGING INVENTORIES

Minimizing total cost (TC) of a fund of cash (or anything else)

Mason Gaffney Notes, 17 May 87, rev. 3/92[1]

Define:

W = monthly cash receipt (W is evidently from "wage")

M = mean amount of money held during month

x = number of transfers into cash holding during month (this is not explicit in H&T, but making it explicit makes things clearer)

k = cost per transfer. (Note, this is independent of the amount transferred. Hence, it represents economies of scale.[2])

R = opportunity interest rate, monthly basis

T = amount of money per deposit

$T = W/x$ (1)

$M = T/2 = W/2x$ (2)

Note that Mx equals a constant, W/2. M and x are inversely related. In general, M symbolizes a fund of capital, x symbolizes turnover, and W/2 symbolizes flow (volume) of funds: remember Volume = Capital x Turnover. (Here, "volume" is another word for "flow.")

Monthly Deposit Cost (DC) = kx (3)

Monthly Carrying Cost (CC) = RM (4)

Monthly Total Cost (TC) = DC + CC = kx + RM (5)

Objective:

Choose a value of M to minimize TC. This is a typical optimizing problem, because it is a trade-off between DC and CC. As M rises, DC falls, and CC rises.

To do this, we must express kx as a function of M. From (2), x=W/2M. Substituting:

$TC = kW/2M + RM$ (6)

Method:

Use differential calculus 1) to find the value of M that minimizes TC, for given values of W, k, and R. (We will denote this value of M as M*); 2) having found M*, to find the optimal value of x that it entails (note, the value of x is not among the three that are given).

Most of you understand something, at least, about calculus. I use little calculus in this course, but here it makes things easier. Differentiate (6), and set its derivative equal to zero, to find

---

1Notation taken from Hall and Taylor, pp. 296-98

2It is like one-stop shopping, or one-pass, large-unit construction. You gain by doing things in one shot; but you pay the price in higher interest cost.

where TC is a max or a min (in this case it is obviously going to be a min).

$$d(TC)/dM = -kW/2M^2 + R = 0$$

$$R = kW/2M^2$$

$$M^* = [kW/2R]^{.5} \qquad (7)$$

(7) is ordinarily called the "square root rule." Let's not be ordinary: that is giving too much emphasis to the square root aspect, which is an artifact of the particular assumptions here. These are arbitrary and specific to the example. The important thing is the sensitivity of M* (optimal M) to k and R. ***M* is an increasing function of k, and a decreasing function of R***.

Figure 1 shows visually, graphically how this works; it is a lot easier to follow and to remember than the calculus, so hold fast to this graph. M is the independent variable here. As M rises, CC rise directly with it, and DC fall. You add these together to get the top curve, TC, which first falls and then rises, giving us a minimum.

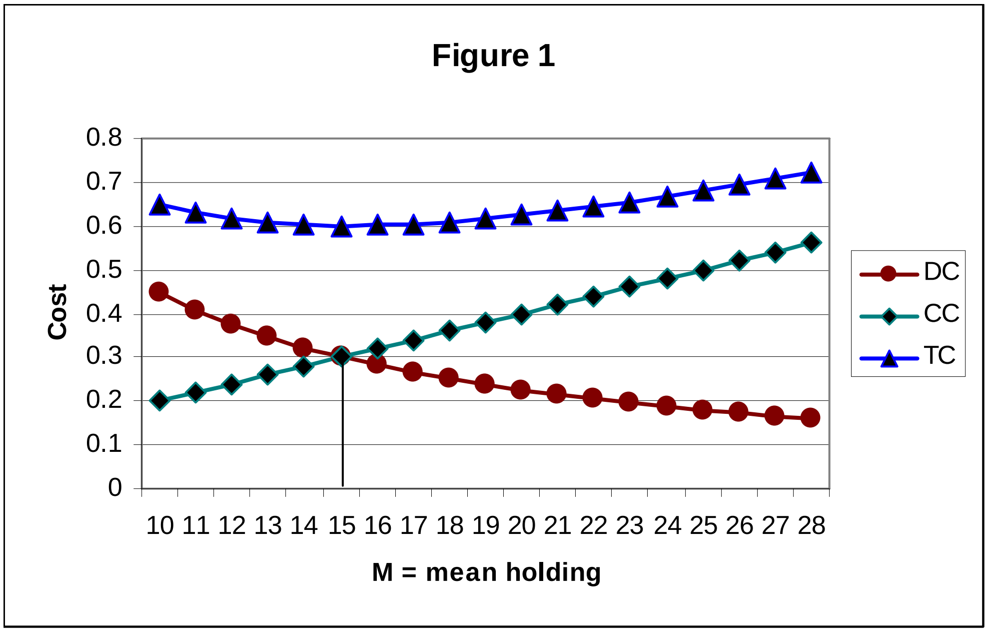

Figure 1: Minimizing TC by adding rising CC and falling DC

TC is minimized where and because the slopes of CC and DC are equal and opposite. It is of incidental interest that their ordinates are equal at this point -- i.e., it is where they cross. As a check, let's see if M has its cost-minimizing value when CC = DC.

$$CC = DC$$

$$RM = kW/2M$$

$$M^2 = kW/2R$$

$$M = [kW/2R]^{.5} \qquad \text{[as in (7)]}$$

That is handy to remember, so remember it. Also remember, however, it is dangerous because you could easily get careless and think that the curves' crossing is a cause of minimizing TC. Not so: beware of this mistake. Like calling it "the square root rule," it is a case of confusing the incidental with the essential. The crossing is the minimum TC in this particular example; not in general.

Now use Figure 1 to observe the effect of R on the optimal holding of cash. Higher R means the CC curve is steeper, moving the min TC to the left, reducing the optimal cash holding.

Eqn. (7) tells you the same thing algebraically: the stock of M that minimizes TC falls as R rises. (This optimal M is M*.)

Use Fig. 1 to observe the effect of k on the optimal holding of cash. Higher k moves the entire DC curve farther away from the origin (up and to the right), moving the crossover to the right, increasing the optimal cash holding.

Eqn. (7) tells you the same thing algebraically: the stock of M which minimizes TC rises as k rises.

What have we shown? If your opportunity cost of money is low, and the value of your time is high, you keep a large cash balance on hand. That is an application of Baumol's inventory model to cash management. However, it is much more general.

SUPPLEMENTAL KNOWLEDGE

Having found optimal M, you can also find optimal x and T, just by substituting the value of optimal M for M. Do this as an exercise. (Where a square root sign covers several terms, just raise each one instead to the .5 power, then combine terms by adding exponents, watching your signs of course as you go.)

You'll find that optimal x is an increasing function of R and a decreasing function of k. This of course is because x, the frequency of trips to the bank, is inversely related to M, the mean stock of money on hand. Learn this well: it has much wider applications, as we emphasize in this course.

1/x is the period of time between trips. Does that ring a bell? We've seen an analogous principle before, when finding the amount of capital in a pipeline. The frequency and the period of anything are always reciprocals.

You'll find that optimal T rises as k rises, and falls as R rises. That is, when you want to economize on your time you do things in large slugs rather than small dribbles; and let the results sit for a long time (1/x) before acting again.

WIDER APPLICATIONS OF THIS MODEL

If we were only interested in determining an optimal stock of cash we wouldn't lean so heavily on this model. But this is also a way to determine an optimal inventory of real capital. Just let M stand for the mean stock of goods in the store; and k for the cost of ordering, delivering and receiving a batch of goods.

It's even more general than that: much more general. Let M stand for the mean life of an item of durable capital; and k stand for the cost of producing same. Now we are moving toward a model of the turnover of all the capital in the nation, not just the inventories.

For now, just note that the incentive to substitute capital (M) for labor (k, in this model) rises as labor costs more. But this does not take the form of a rise in the flow of investment, but the reverse, because x falls as k rises.